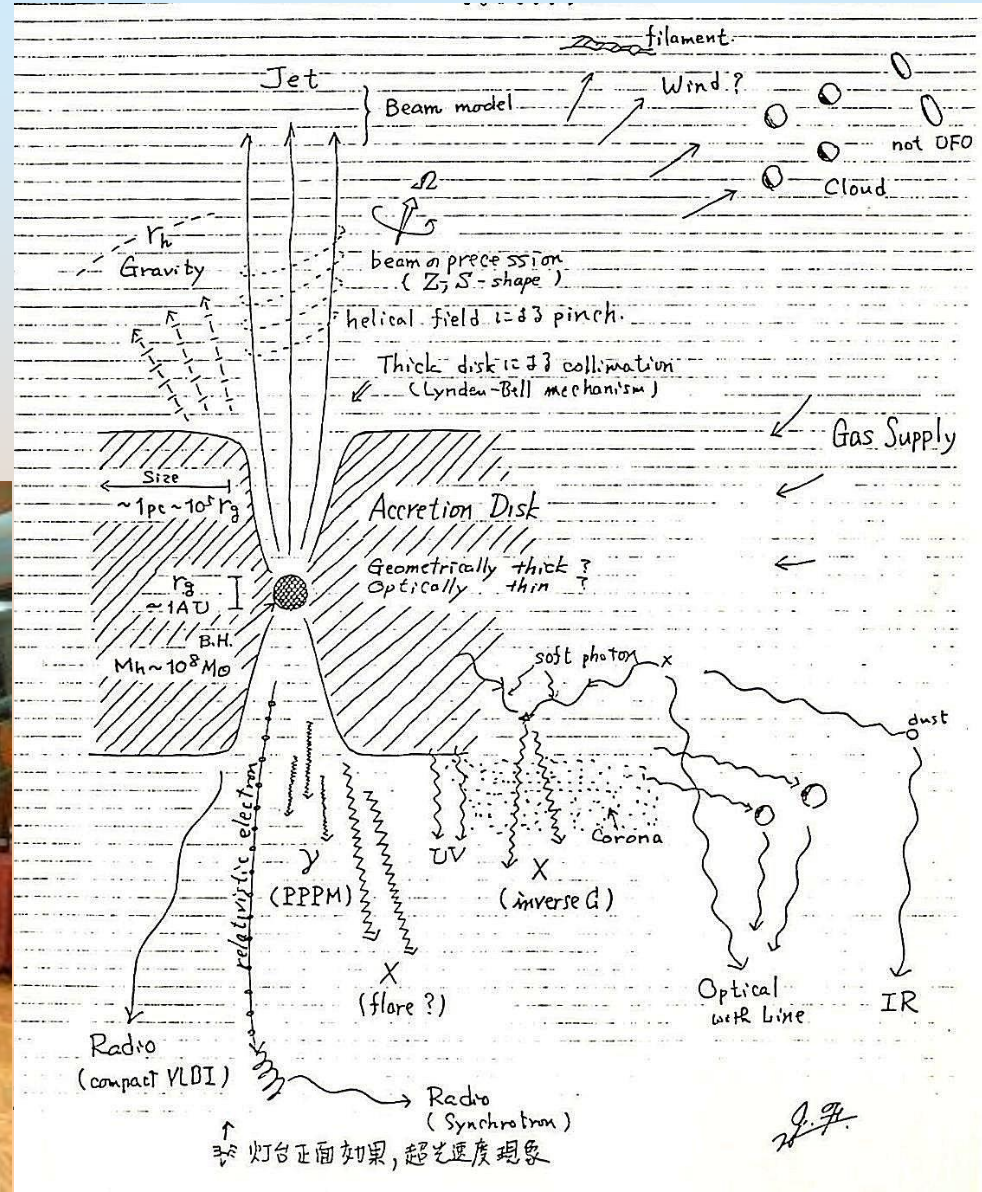
降着流の基礎

1. Eddington Luminosity
2. Hoyle-Lyttleton Accretion
3. Bondi Accretion
4. Transonic Nature

Kyoto Prize 2011

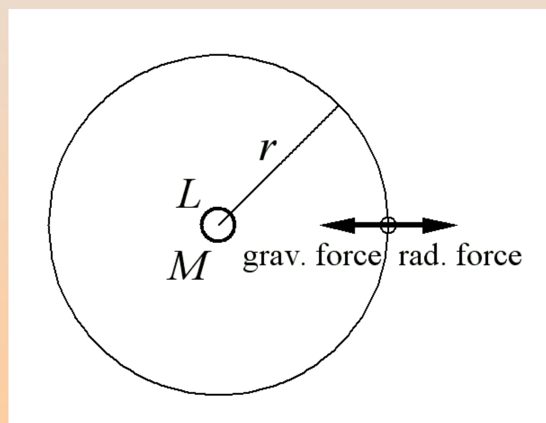


0　エディントン光度

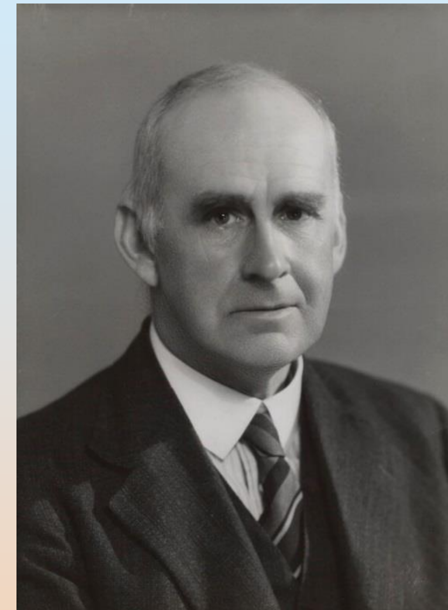
- 重力＝輻射圧となる天体の“最大”光度
- 光学的に薄い場合
  - 重力
  - 輻射圧
    - 天体の質量M
    - 水素原子の質量m_{H}
    - 距離r
    - トムソン散乱の断面積σ_{T}

$$F_{\mathrm{g}} = \frac{GMm_{\mathrm{H}}}{r^2},$$

$$\frac{\sigma_{\mathrm{T}}}{c} f = \frac{\sigma_{\mathrm{T}}}{c} \frac{L}{4\pi r^2}.$$



- 光学的に厚い場合
  - エディントン大気
  - $\kappa L_r/M_r$＝一定
  - $\beta = p/(p+P)$＝一定



Sir Arthur Eddington
(1882-1944)

エディントン光度
臨界降着率

- エディントン光度L_E（Eddington luminosity）

$$L_\mathrm{E} = \frac{4\pi c G M m_\mathrm{H}}{\sigma_\mathrm{T}}.$$
$$= 1.25 \times 10^{39} \left(\frac{M}{10 M_\odot}\right) \ \mathrm{erg\ s^{-1}}$$
$$= 1.25 \times 10^{46} \left(\frac{M}{10^8 M_\odot}\right) \ \mathrm{erg\ s^{-1}}$$

- エディントン降着率
  - 効率η～0.1
  - “最大”降着率（球対称）
  - 臨界質量降着率（critical accretion rate）

$$\dot{M}_\mathrm{crit} \equiv \frac{L_\mathrm{E}}{c^2} = 1.4 \times 10^{17} \frac{M}{M_\odot} \ \mathrm{g\ s^{-1}}.$$

$$\dot{M} \lesssim \frac{1}{\eta} \frac{L_\mathrm{E}}{c^2}.$$

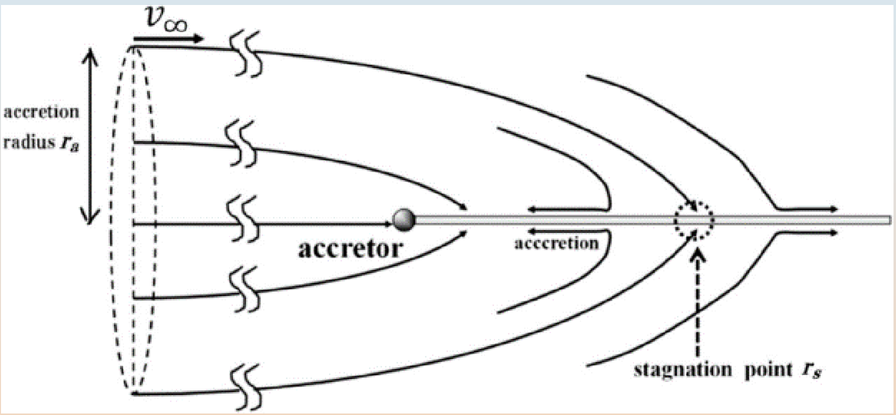
- エディントン時間
  - 質量に依存しない

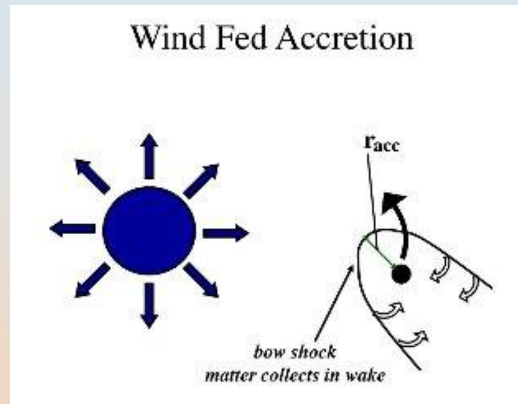
$$t_\mathrm{E} \equiv \frac{M}{\dot{M}_\mathrm{crit}} = \frac{\sigma_\mathrm{T} c}{4\pi G m_\mathrm{H}} = 4.5 \times 10^8 \ \mathrm{yr}.$$

1　ホイル＝リットルトン降着

Hoyle and Lyttleton 1939

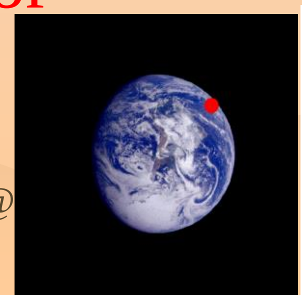
- 重力天体への質量降着の基本機構

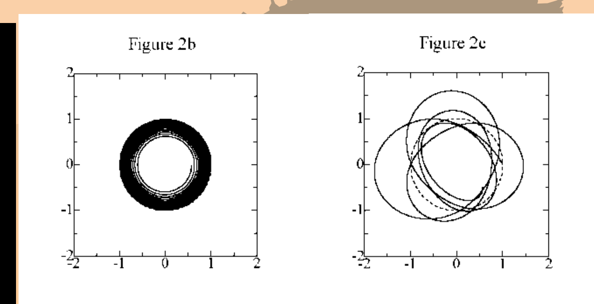




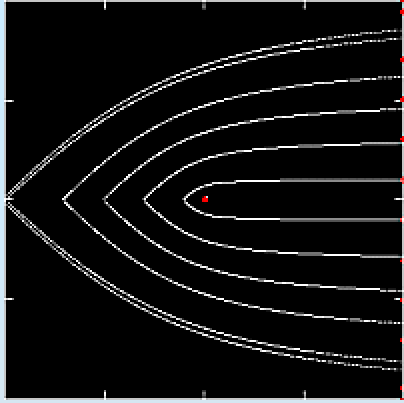
- 天文学的事象
  - interstellar <u>moving</u> accretor
  - wind fed accretion
  - micro BH sydrome

Sir Fred Hoyle (1915-2001)
Raymond Arthur Lyttleton (1911-1995)

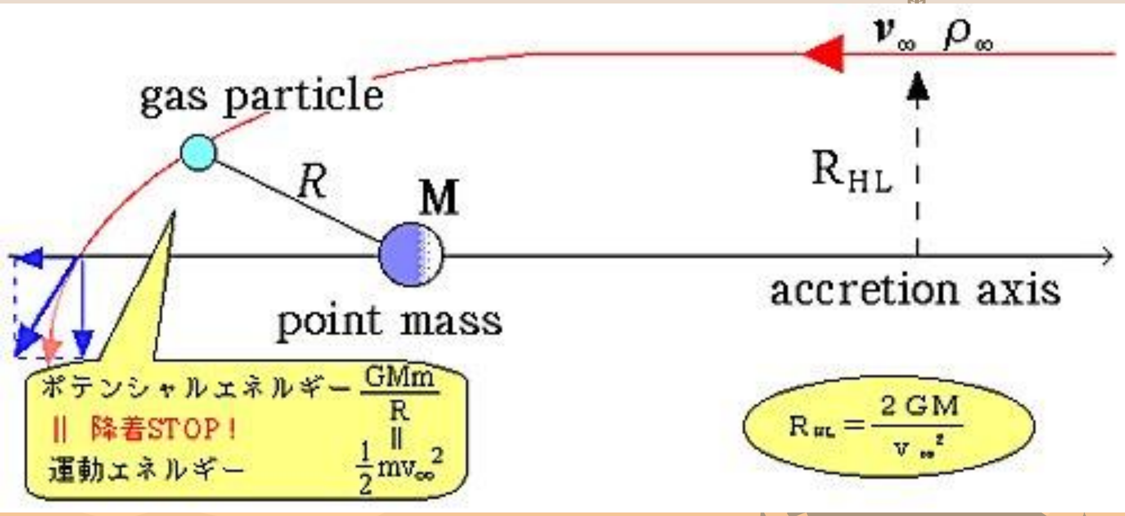




ホイル＝リットルトン降着仮定

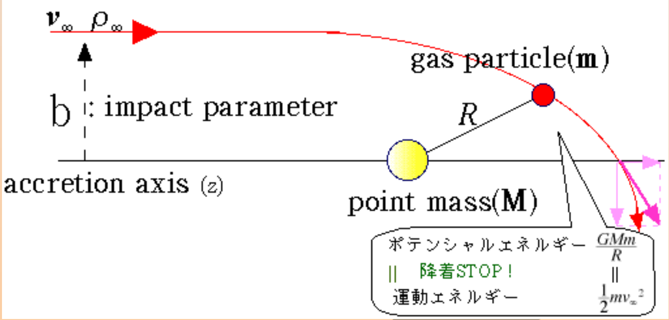


- 軸対称降着（動いている）
  - 質量Mの降着体、速度v_∞、一様密度ρ_∞
  - 定常
  - 軸対称（降着軸）
  - 非粘性
  - 断熱
  - 磁場・輻射なし



ホイル＝リットルトン降着
粗い見積もり

- 中心天体に対するガス粒子の相対速度をv、衝突パラメータをbとする
  - ガスの運動エネルギー～$v^2/2$
  - ガスの重力エネルギー～GM/b
- ガスが束縛される条件：等置
  - $R_{\mathrm{HL}}=2GM/v^2$



- 軌道計算による精密な見積もりも同じになる

ホイル＝リットルトン降着
降着半径など

- ホイル＝リットルトン降着半径R_{HL}
- ホイル＝リットルトン降着タイムスケールt_{HL}
- ホイル＝リットルトン質量降着率$\dot{M}_{HL}$＝$\rho v_\infty \pi R_{HL}{}^2$
- ホイル＝リットルトン成長時間t_{growth}＝M／$\dot{M}_{HL}$

$$R_{\rm HL} = \frac{2GM}{v_\infty^2} = 2.65\times10^{15}\,\frac{M}{10\,M_\odot}\left(\frac{v_\infty}{10\ {\rm km\ s^{-1}}}\right)^{-2}\ {\rm cm.}$$

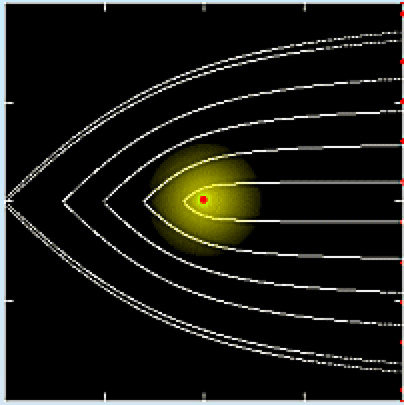
$$t_{\rm HL} = \frac{R_{\rm HL}}{v_\infty} = 84.0\,\frac{M}{10\,M_\odot}\left(\frac{v_\infty}{10\ {\rm km\ s^{-1}}}\right)^{-3}\ {\rm yr.}$$

$$\dot{M}_{\rm HL} = \pi R_{\rm HL}^2\rho_\infty v_\infty = \frac{4\pi\rho_\infty G^2M^2}{v_\infty^3} = 3.70\times10^{18}\left(\frac{M}{10\,M_\odot}\right)^2\frac{n_\infty}{10^5\ {\rm cm^{-3}}}\left(\frac{v_\infty}{10\ {\rm km\ s^{-1}}}\right)^{-3}\ {\rm g\ s^{-1}},$$

$$t_{\rm growth} = \frac{M}{\dot{M}_{\rm HL}} = 1.70\times10^{8}\left(\frac{M}{10\,M_\odot}\right)^{-1}\left(\frac{n_\infty}{10^5\ {\rm cm^{-3}}}\right)^{-1}\left(\frac{v_\infty}{10\ {\rm km\ s^{-1}}}\right)^{3}\ {\rm yr.}$$

HL : 輻射圧の効果

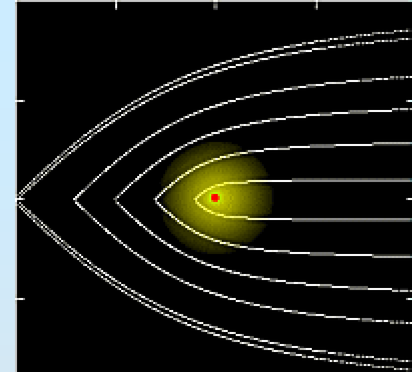
Taam+ 1991



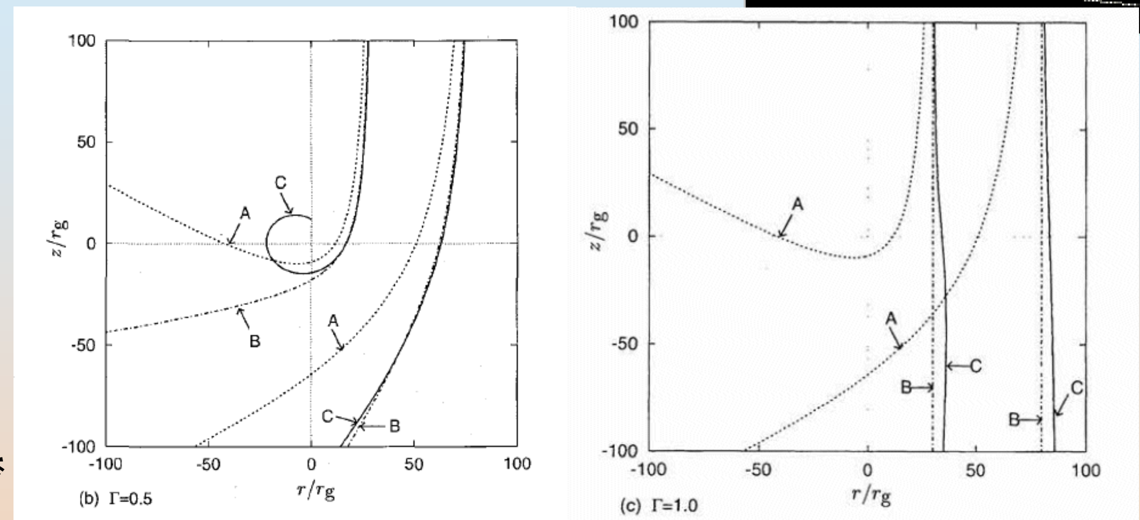
- エディントン比　$\Gamma=L/L_E$
- 光学的に薄い
- $M \rightarrow (1-\Gamma)M$
- 降着半径R_{HL}
- $R_{HL}{}^{rad}=(1-\Gamma)R_{HL}$
- 降着タイムスケールt_{HL}
- $t_{HL}{}^{rad}=(1-\Gamma)t_{HL}$
- 質量降着率$\dot{M}_{HL}=\rho v \pi R_{HL}{}^2$
- $\dot{M}_{HL}{}^{rad}=(1-\Gamma)^2 \dot{M}_{HL}$
- 成長時間$t_{growth}=M / \dot{M}_{HL}$
- $t_{growth}{}^{rad}=t_{growth}/(1-\Gamma)^2$

HL：輻射抵抗の効果

Nio+ 1998

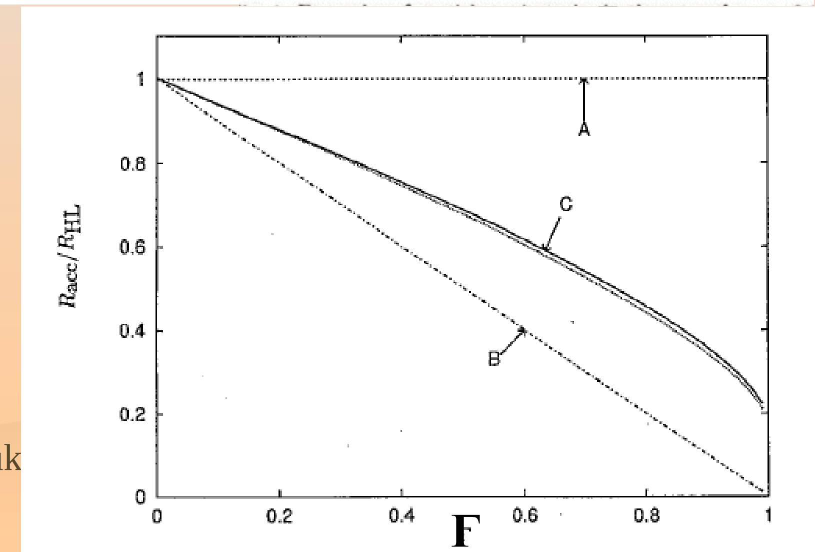


- エディントン比$\Gamma=L/L_E$
- 光学的に薄い
- 輻射抵抗 $\propto$ v
- $O(v/c)^1$



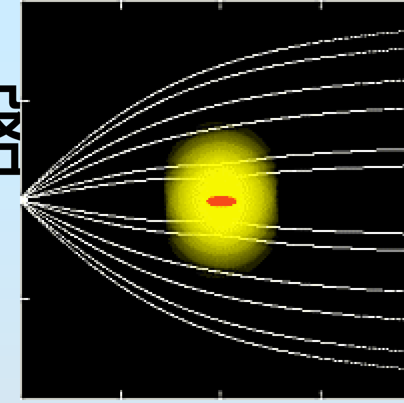
A：重力のみ
B：輻射入り
C：輻射抵抗まで
$\Gamma=0.5,\ 1.0$

- 降着半径R_{HL}
- $R_{acc}^{drag}=[1-\Gamma(1-2v_\infty/c)]R_{HL}$



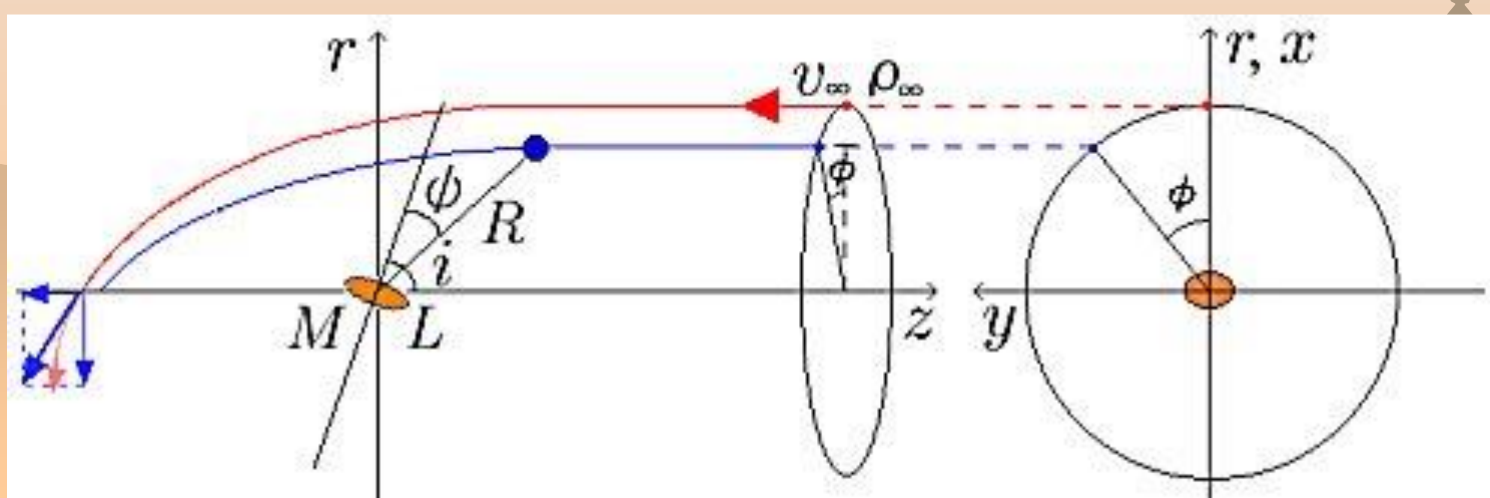
HL : 中心天体が降着円盤

JF+Ioroi 1999

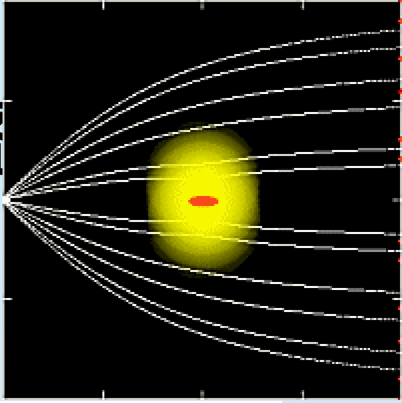


- エディントン比 $\Gamma = L/L_{\mathrm{E}}$
- 光学的に薄い
- 中心に降着円盤
- 光源が非等方

$$F = \frac{L}{2\pi R^2} \cos\theta,$$



HL : 中心天体が降着円盤



JF+Ioroi 1999

- 輻射場の分布
  - 降着円盤対称軸　i
  - ガス粒子の方向　s
  - なす角度　ψ
  - 動径輻射流束　F_R
  - 円筒座標での成分　F_r, F_z

$$\boldsymbol{i} = (\sin i, 0, \cos i)\,,$$
$$\boldsymbol{s} = \left(\frac{r}{R}\cos\varphi, \frac{r}{R}\sin\varphi, \frac{z}{R}\right).$$

$$\cos\psi = \boldsymbol{i}\cdot\boldsymbol{s} = \frac{1}{R}(r\cos\varphi\sin i + z\cos i)\,.$$

$$F_R = \frac{L\cos\psi}{2\pi R^2}.$$

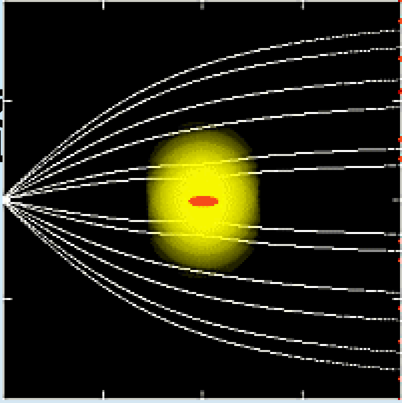
$$F_r = F_R\frac{r}{R},$$
$$F_z = F_R\frac{z}{R}.$$

- 運動方程式
  - 円筒座標での成分
  - 有効規格化光度（場所の関数）

$$\frac{dv_r}{dt} = -\frac{GMr}{R^3} + \frac{\sigma_T}{mc}F_R\frac{r}{R}, \quad = -\frac{GMr}{R^3}(1-\Gamma_{\mathrm{eff}}),$$
$$\frac{dv_z}{dt} = -\frac{GMz}{R^3} + \frac{\sigma_T}{mc}F_R\frac{z}{R}. \quad = -\frac{GMz}{R^3}(1-\Gamma_{\mathrm{eff}}),$$

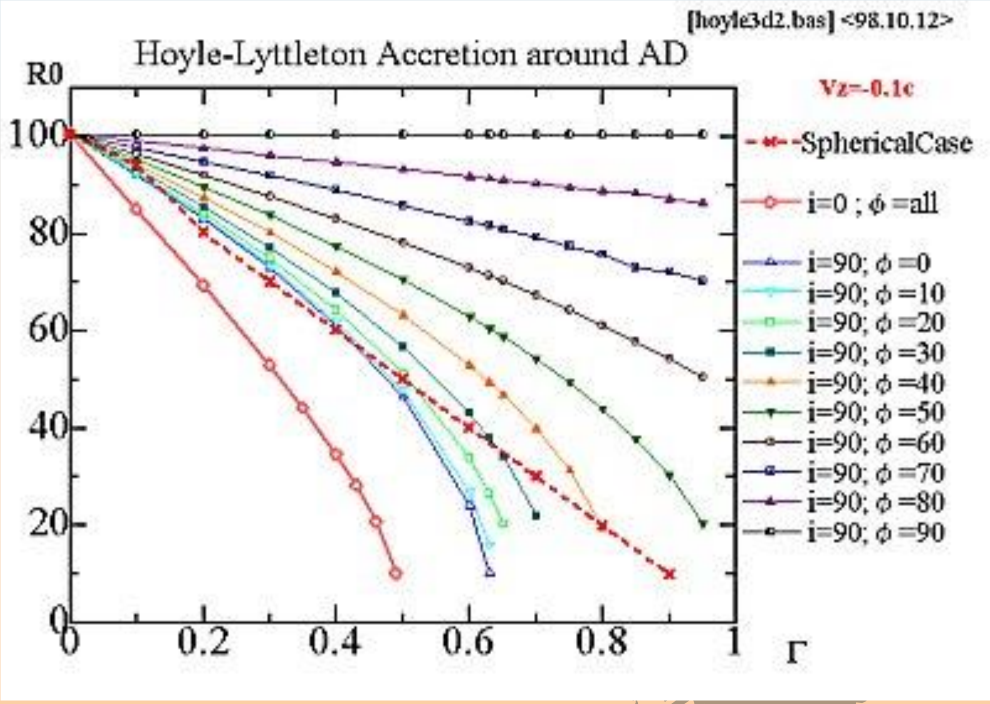
$$\Gamma_{\mathrm{eff}} \equiv 2\Gamma\cos\psi$$
$$= 2\frac{L}{L_E}\frac{r\cos\varphi\sin i + z\cos i}{R}.$$

HL：中心天体が降着円盤
JF+Ioroi 1999

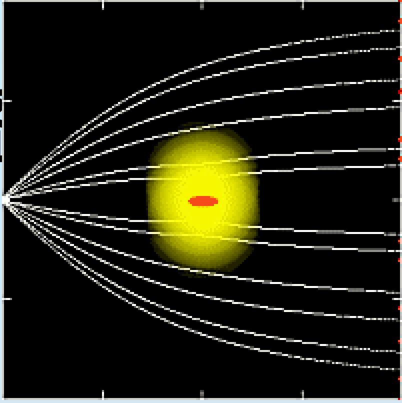


- 計算結果：降着半径

- 降着半径Racc/RHLー 規格化光度Γ
- 点線：球対称な場合
  - （1ーΓ）に比例して降着半径は小さくなる．
- 実線：pole-on降着（i＝0）
  - 降着円盤の輻射場は円盤に鉛直方向に強くなっている（正面から見たら降着円盤は明るく見える）．その結果，流入するガスに対して円盤面が正面を向くpole-on降着では，ガスの受ける輻射圧はより強くなるので，図のように，球対称な場合より，降着半径はさらに小さくなるのである．
- 多数の実線：edge-on降着（i＝90°）
  - 実線の違いは降着軸のまわりの円周角φに対応する．このedge-on降着では，降着半径がφに強く依存するので複雑だが，大ざっぱには，縁方向から見れば降着円盤は暗く見える．その結果，輻射圧の効果は弱まり，球対称な場合より，降着半径は大き目になる．



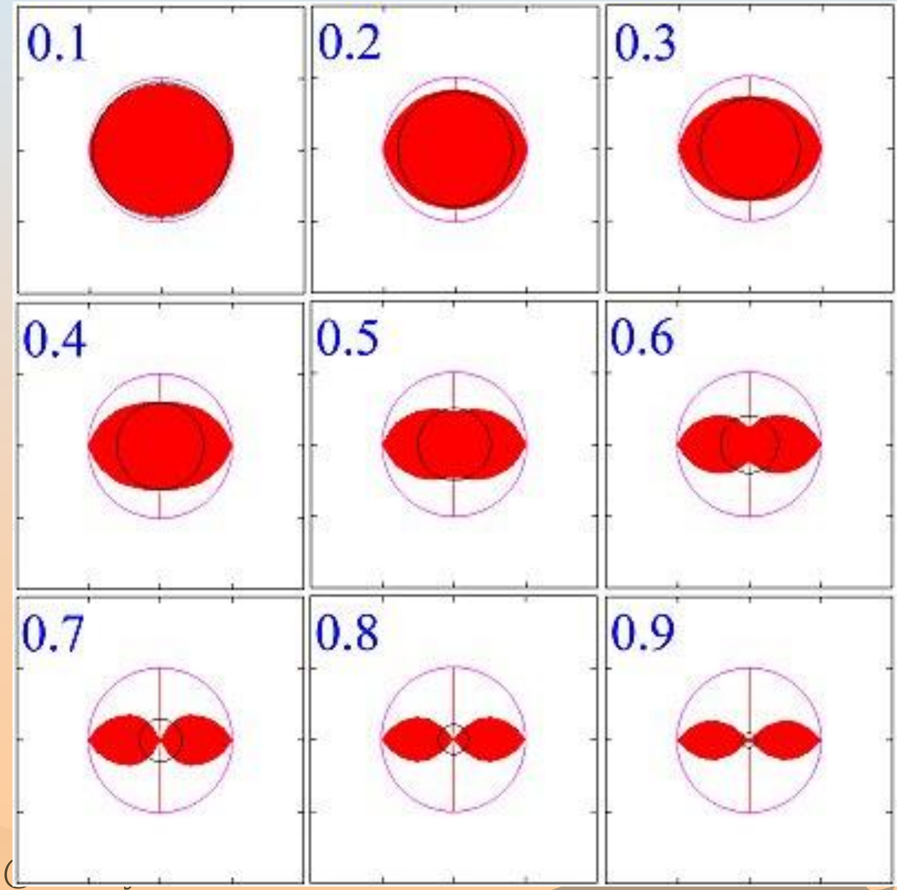
HL：中心天体が降着円盤



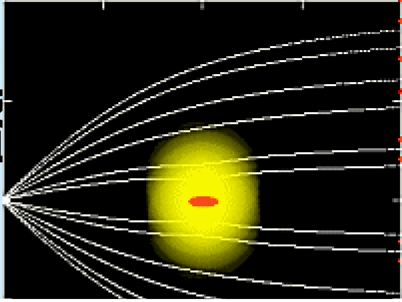
JF+Ioroi 1999

- 計算結果：降着領域の断面積

- edge-on降着（i＝90°）（数字はΓ）
  - 外側の円は古典的なホイル＝リットルトン降着半径RHLで，内側の円は球対称な場合の降着半径を表す．
  - 円盤面内（φ＝90°；図の左右）で入射する粒子は，降着円盤からの輻射を受けないので，降着半径は古典的な半径RHLに一致する．一方，円盤の上方（φ＜90°；図の上下方向）で入射する粒子は，降着円盤の輻射場の影響を受けて，降着半径は小さくなる．その結果，降着領域の断面積は，図のように上下につぶれたものになる．
  - より細かく言えば，降着領域の断面積は，Γが小さいと上下に潰れた楕円形になり，Γが0.5ぐらいで上下に凹んだ形状になり，Γが0.65ぐらいからは真上方向では降着できなくなるため，双葉のような形状になる．



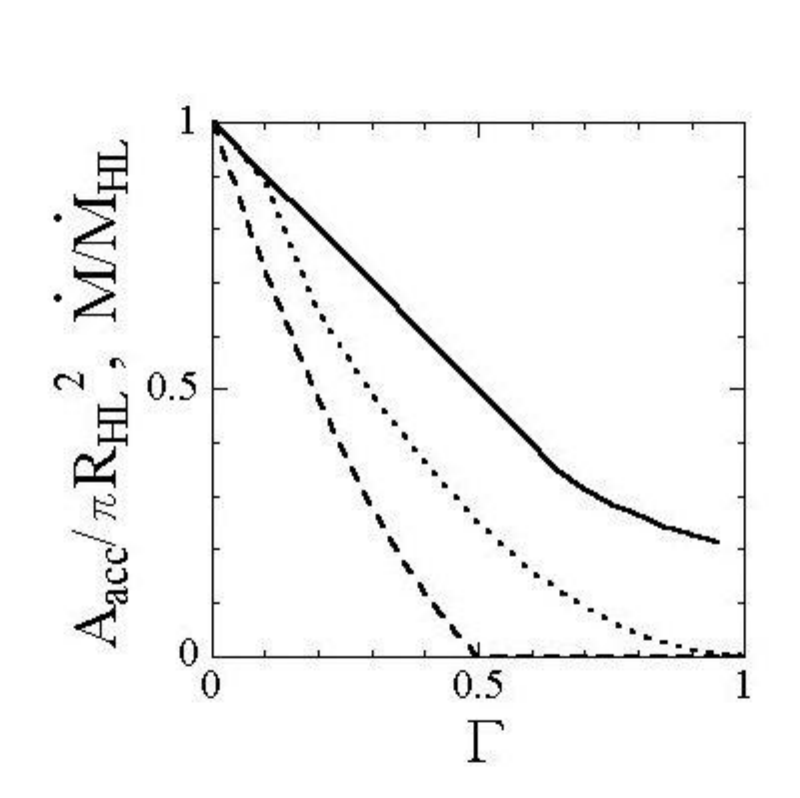
HL：中心天体が降着円盤 JF+Ioroi 1999



- 計算結果：質量降着率

- 降着断面積ー光度Γ
- ホイル＝リットルトン降着の質量降着率で規格化した質量降着率

- 点線：球対称な場合
  - （1ーΓ）2に比例して小さくなる.
- 破線：pole-on降着（i＝0）の場合
  - 球対称の場合より小さい.
- 実線：edge-on降着（i＝90°）の場合
  - 球対称の場合より大きい.
  - このedge-on降着の場合で特筆すべきことは，規格化した光度Γが1のとき（降着円盤の光度がエディントン光度に等しいとき）でも，質量降着が可能な点である.
- 上の計算結果を近似的に表した式



$$A_{acc} = \pi R_{HL}^2 f(\Gamma, i),$$

$$f(\Gamma, i) = \begin{cases} (1-\Gamma)^2 & \text{for a spherical case} \\ (1-\Gamma)(1-2\Gamma) & \text{for } i = 0 \\ (1-\Gamma) & \text{for } i = 90^\circ, \Gamma \le 0.65 \\ (2-\Gamma)^2/5 & \text{for } i = 90^\circ, \Gamma \ge 0.65. \end{cases}$$

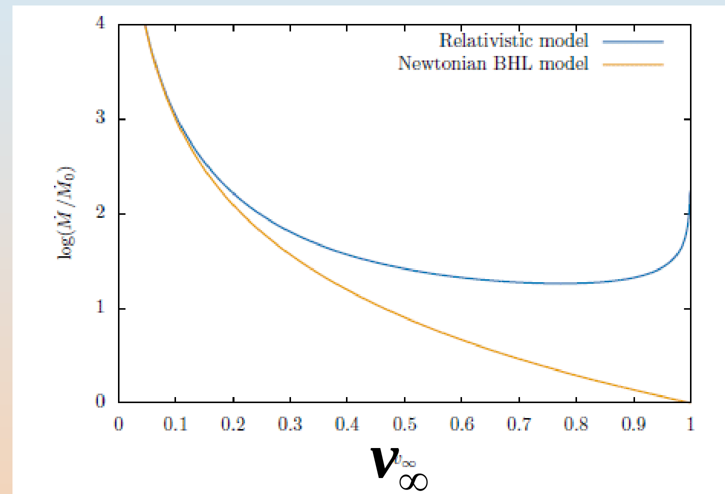
最近の話題と今後の課題

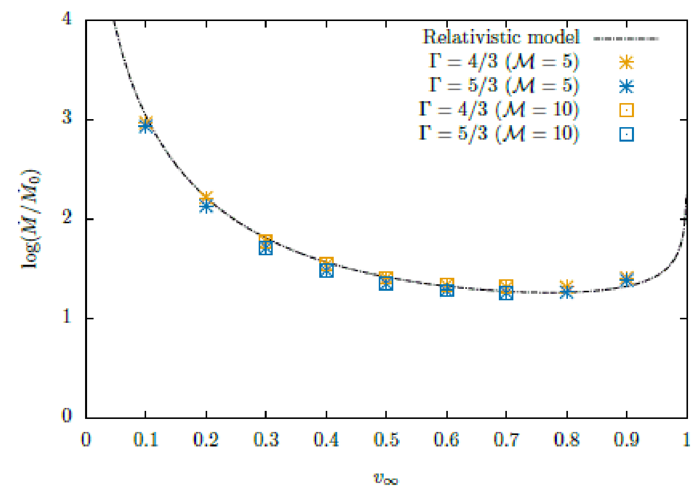
- radiation field, optical depth
- magnetic field
- relativistic effect
- simulation
- wave
- stability
- multi-components

最近の話題と今後の課題

Tejeda and Aguayo-Ortiz 2019

- relativistic: Sch. BH
- ballistic model
- analytical/simulation
- 降着率が大きく増加





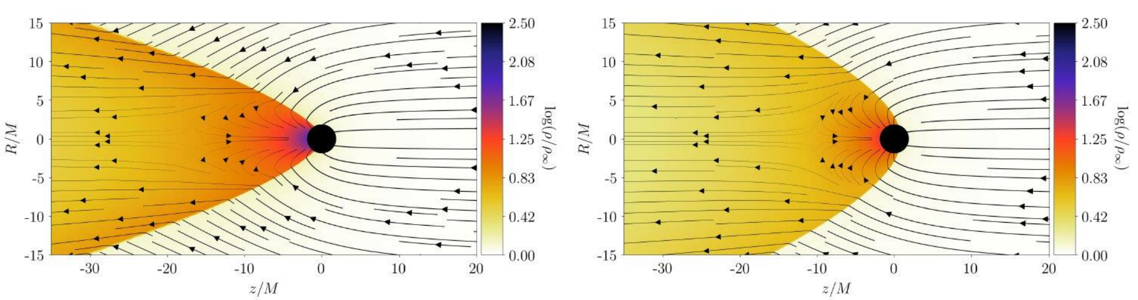
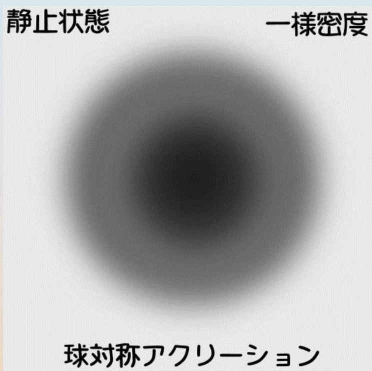


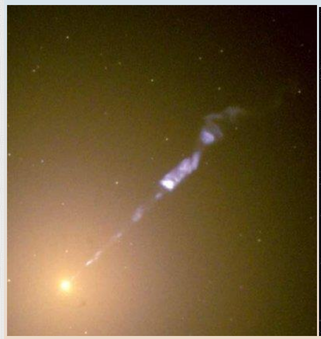
Figure 7. Numerical simulations performed with *aztekas* of a relativistic wind accreting onto a Schwarzschild black hole. The chosen polytropic index is $\Gamma = 4/3$ for the left side panel and $\Gamma = 5/3$ for the right side one, while the asymptotic wind velocity is $v_\infty = 0.5$. The plot shows the stationary-state streamlines and in colour isocontour levels of the corresponding density field. The width of the streamlines is proportional to the velocity magnitude $v = \sqrt{v_i\, v^i}$.

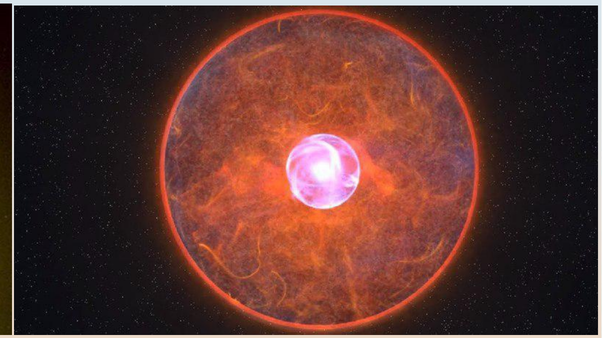
2　ボンヂ降着

Bondi 1952

- 重力天体への球対称降着の基本モデル





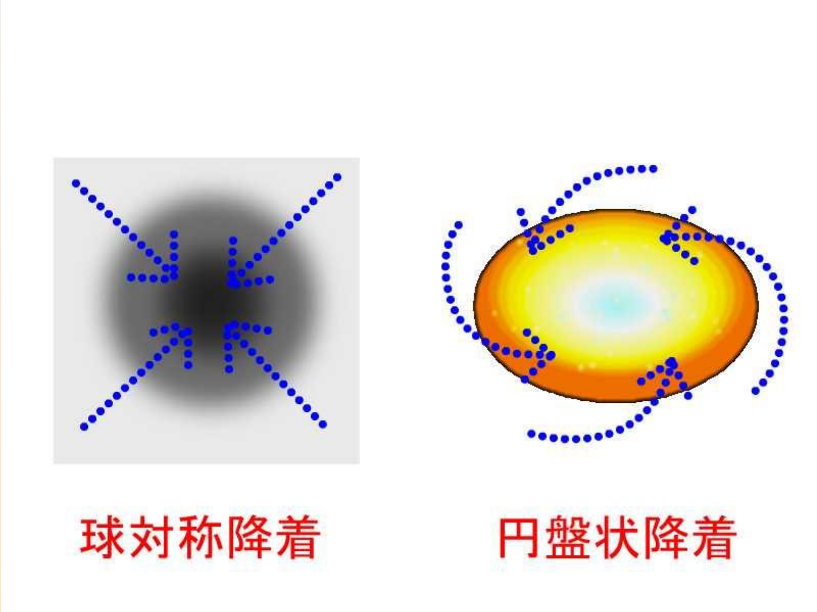


Sir Hermann Bondi (1919-2005)

- 天文学的事象
  - interstellar static accretor
  - young stellar object
  - AGN/adaf-like SMBH/M87
  - Thorne-Zytkow object

ボンヂ降着
仮定

- 球対称降着
  - 質量Mの中心天体、一様密度ρ_∞
  - 定常
  - 球対称(r)
  - 非粘性
  - 断熱
  - 磁場・輻射なし



ボンヂ降着
基礎方程式

- 連続の式
  - 密度ρ、速度v
- 運動方程式
  - 圧力p
- 状態方程式
  - 比熱比γ
- エネルギー積分
  - ベルヌーイ定数E

$$\frac{1}{4\pi r^2}\frac{d}{dr}\left(4\pi r^2 \rho v\right) = 0,$$

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM}{r^2},$$

$$p = K\rho^\gamma,$$

$$-4\pi r^2 \rho v = \dot{M},$$

$$\frac{1}{2}v^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} - \frac{GM}{r} = E,$$

ボンヂ降着
風方程式

- 流速と音速
  - 音速$c_s{}^2=dp/d\rho$
- 積分形

$$-4\pi r^2 c_s^{\frac{2}{\gamma-1}} v = (K\gamma)^{\frac{1}{\gamma-1}} \dot{M},$$

$$\frac{1}{2}v^2 + \frac{1}{\gamma-1}c_s^2 - \frac{GM}{r} = E.$$

- 微分形：風方程式

$$(v^2 - c_s^2)\frac{1}{v}\frac{dv}{dr} = \frac{2}{r}c_s^2 - \frac{GM}{r^2}.$$

$$c_s^2 = (\gamma - 1)\left(E + \frac{GM}{r} - \frac{1}{2}v^2\right).$$

ボンヂ降着
風方程式

- マッハ数
  - マッハ数 $M = v/c_s$
  - 臨界点（critical point）／遷音速点（transonic point）: $D=0=N$

$$\frac{d\mathcal{M}}{dr} = \frac{\mathcal{N}}{\mathcal{D}}, \tag{2.29}$$

$$\mathcal{D} = \mathcal{M}^2 - 1, \tag{2.30}$$

$$\mathcal{N} = \mathcal{M}\left(\frac{\gamma-1}{2}\mathcal{M}^2+1\right)\left[\frac{2}{r} - \frac{\gamma+1}{2(\gamma-1)}\frac{1}{E+\frac{GM}{r}}\frac{GM}{r^2}\right], \tag{2.31}$$

ボンヂ降着
特異点

- 特異点解析
  - 風方程式の分母Dと分子Nを、それぞれ、臨界点(特異点)の近傍で2変数で線形展開する
  - 展開係数は臨界点の値で評価する

$$\mathcal{D}(r,\mathcal{M}) \sim \mathcal{D}|_c + \left.\frac{\partial \mathcal{D}}{\partial r}\right|_c dr + \left.\frac{\partial \mathcal{D}}{\partial \mathcal{M}}\right|_c d\mathcal{M},$$

$$\mathcal{N}(r,\mathcal{M}) \sim \mathcal{N}|_c + \left.\frac{\partial \mathcal{N}}{\partial r}\right|_c dr + \left.\frac{\partial \mathcal{N}}{\partial \mathcal{M}}\right|_c d\mathcal{M}.$$

$$\begin{aligned}
\lambda_{11} &\equiv \left.\frac{\partial \mathcal{D}}{\partial r}\right|_c = 0, \\
\lambda_{12} &\equiv \left.\frac{\partial \mathcal{D}}{\partial \mathcal{M}}\right|_c = 2, \\
\lambda_{21} &\equiv \left.\frac{\partial \mathcal{N}}{\partial r}\right|_c = \frac{5-3\gamma}{r_c^2}, \\
\lambda_{22} &\equiv \left.\frac{\partial \mathcal{N}}{\partial \mathcal{M}}\right|_c = 0
\end{aligned}$$

ボンチ降着
特異点

- 特異点解析
  - 風方程式に展開を代入すると、dM/drに関する2次方程式が得られる
  - この方程式は臨界点のごく近傍だけで成り立つ方程式で、2つの解をもつ

$$\frac{d\mathcal{M}}{dr} = \frac{\lambda_{21} dr + \lambda_{22} d\mathcal{M}}{\lambda_{11} dr + \lambda_{12} d\mathcal{M}} = \frac{\lambda_{21} + \lambda_{22} \dfrac{d\mathcal{M}}{dr}}{\lambda_{11} + \lambda_{12} \dfrac{d\mathcal{M}}{dr}}.$$

$$\lambda_{12} \left(\frac{d\mathcal{M}}{dr} \right)^2 + (\lambda_{11} - \lambda_{22}) \frac{d\mathcal{M}}{dr} - \lambda_{21} = 0,$$

$$\frac{d\mathcal{M}}{dr} = \frac{-\lambda_{11} + \lambda_{22} \pm \sqrt{(\lambda_{11} - \lambda_{22})^2 + 4\lambda_{12}\lambda_{21}}}{2\lambda_{12}}.$$

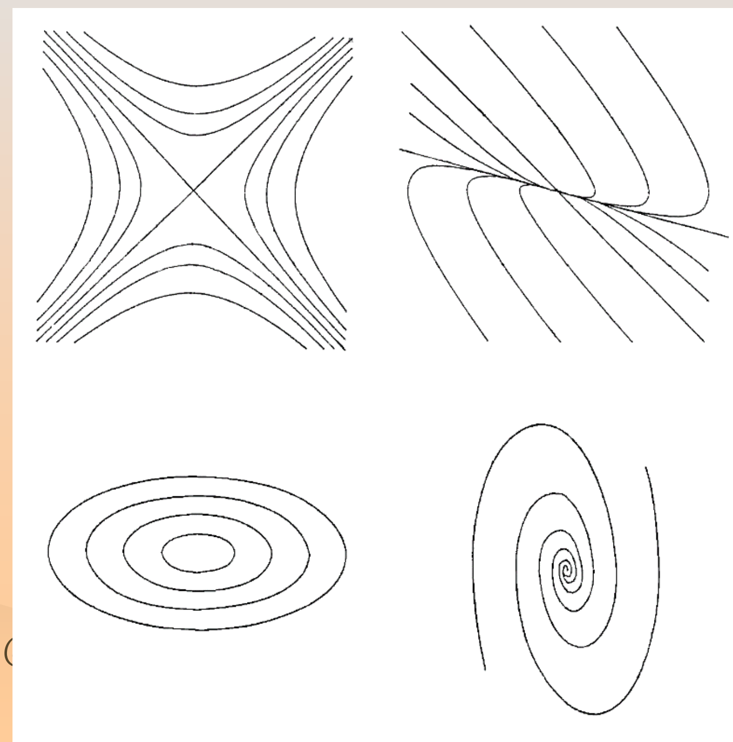
$$\frac{d\mathcal{M}}{dr} = \pm \sqrt{\frac{5 - 3\gamma}{2r_c^2}}.$$

ボンヂ降着
トポロジー

- 臨界点のトポロジー
  - 鞍点saddle
  - 結節点node
  - 渦心点center/spiral
- 非粘性、断熱
  - 鞍点
- 粘性、熱伝導
  - 結節点
  - But　causality problem

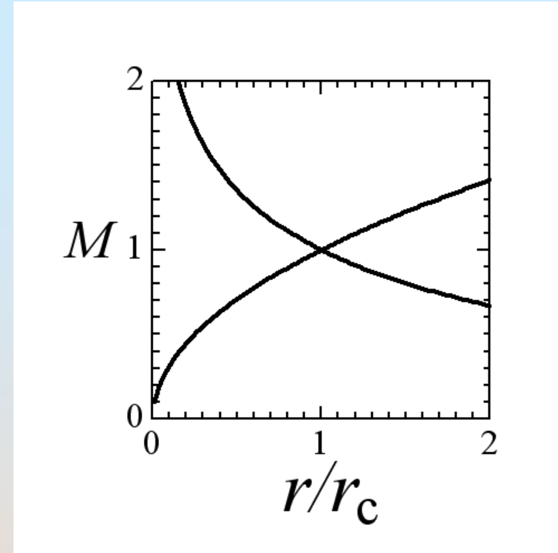
Table 2.1 Roots and Types of Critical Points.

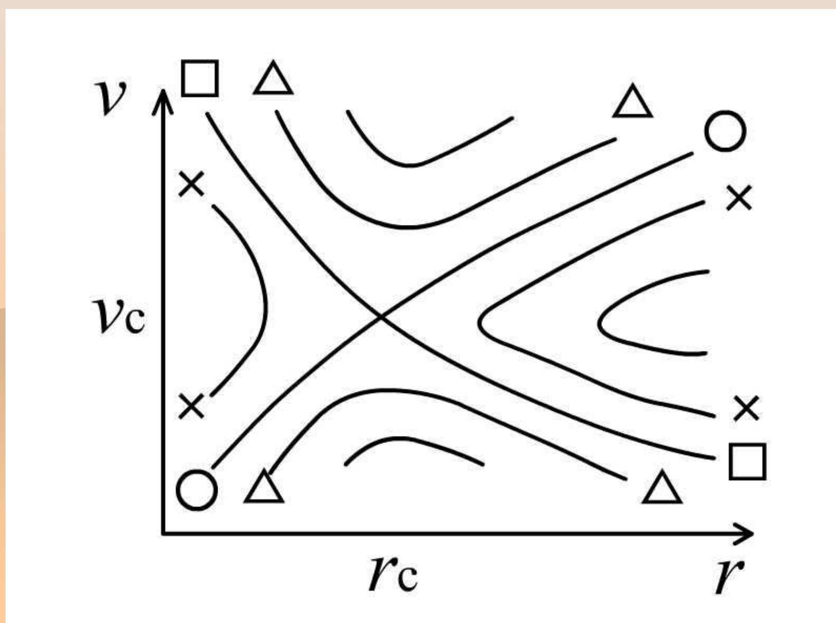
Roots	Type
Two real roots with opposite sign	saddle
Two real roots with same sign	node
Pure imaginary roots	center
Complex roots	spiral

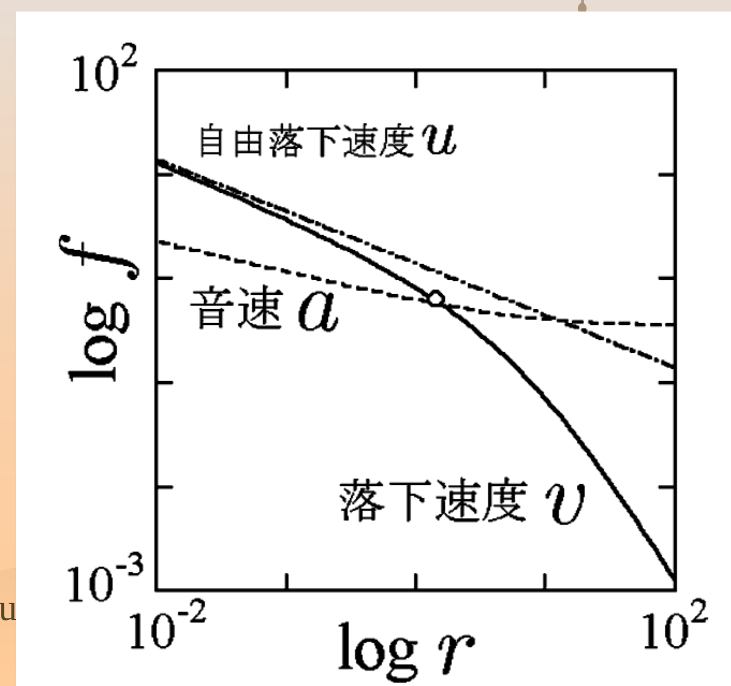


ボンヂ降着
定常解

- 風解（外向き）
  - Parker 1964
- 降着解（内向き）
  - Bondi 1957







ボンヂ降着
降着半径など

- ボンヂ降着半径R_B

$$\frac{GM}{c_{s\infty}^2} = 7.98 \times 10^{14}\ \mathrm{cm} \times \gamma^{-1} \left(\frac{M}{10\ M_\odot}\right) \left(\frac{T_\infty}{10^4\ \mathrm{K}}\right)^{-1}$$

- ボンヂ質量降着率
$\dot{M}_B = \rho c_s \pi R_B{}^2$

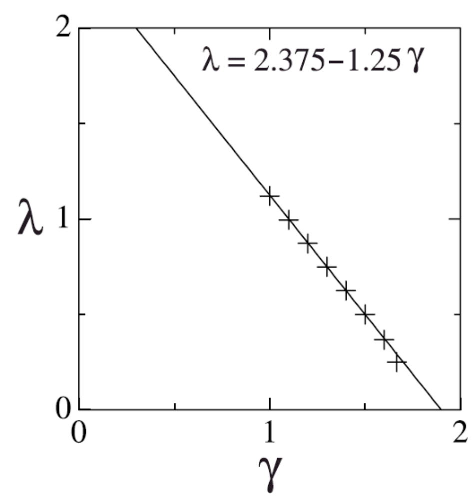
$$\dot{M}_B \equiv \lambda(\gamma) \frac{4\pi (GM)^2 \rho_\infty}{c_{s\infty}^3}$$
$$= 2.74 \times 10^{-13}\ M_\odot\ \mathrm{yr}^{-1} \lambda(\gamma) \gamma^{-3/2} \times \left(\frac{M}{10\ M_\odot}\right)^2 \left(\frac{T_\infty}{10^4\ \mathrm{K}}\right)^{-3/2} \left(\frac{n_\infty}{1\ \mathrm{cm}^{-3}}\right).$$

- rigorous（Bondi 1952）

$$\lambda(\gamma) = \frac{1}{4}\left(\frac{2}{5-3\gamma}\right)^{\frac{5-3\gamma}{2(\gamma-1)}}$$

- approximate（Fukue 2001b）

$$\lambda(\gamma) = -\frac{5}{4}\gamma + \frac{19}{8}$$



Bondi：輻射圧の効果：球、円盤
Fukue 2001b

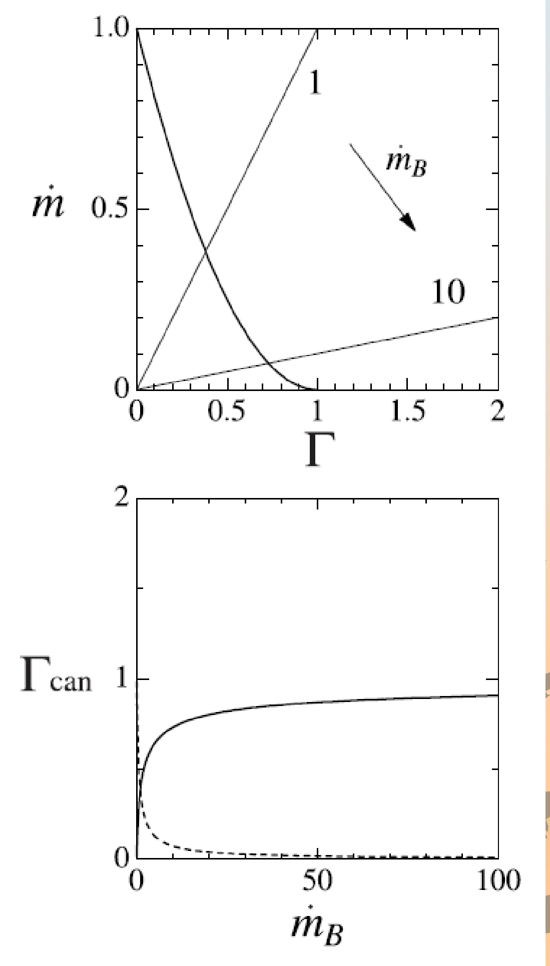
- 質量降着率general
- $\dot{M}_B{}^{rad} = (1-\Gamma)^2 \dot{M}_B$
- Accretion Luminosity

$$\frac{\dot{M}}{\dot{M}_B} = \frac{\dot{M}_E}{\dot{M}_B}\Gamma = \frac{1}{\dot{m}_B}\Gamma, \quad \dot{m}_B \equiv \frac{\dot{M}_B}{\dot{M}_E} = \frac{\eta \dot{M}_B c^2}{L_E}.$$

$$\dot{M}_E = 2.21 \times 10^{-7}\, M_\odot\, \mathrm{yr}^{-1} \left(\frac{\eta}{0.1}\right)^{-1} \left(\frac{M}{10\, M_\odot}\right)^{2}$$

$$\dot{m}_B = 0.124\lambda\gamma^{-3/2} \left(\frac{\eta}{0.1}\right)^{-1} \left(\frac{M}{10\, M_\odot}\right) \left(\frac{T_\infty}{10^4\,\mathrm{K}}\right)^{-3/2} \left(\frac{n_\infty}{10^5\,\mathrm{cm}^{-3}}\right)$$

$$\Gamma_{\mathrm{can}} = 1 + \frac{1}{2\dot{m}_B} - \sqrt{\left(1 + \frac{1}{2\dot{m}_B}\right)^2 - 1}.$$



Bondi：ガス-ダスト
Fukue 2001a

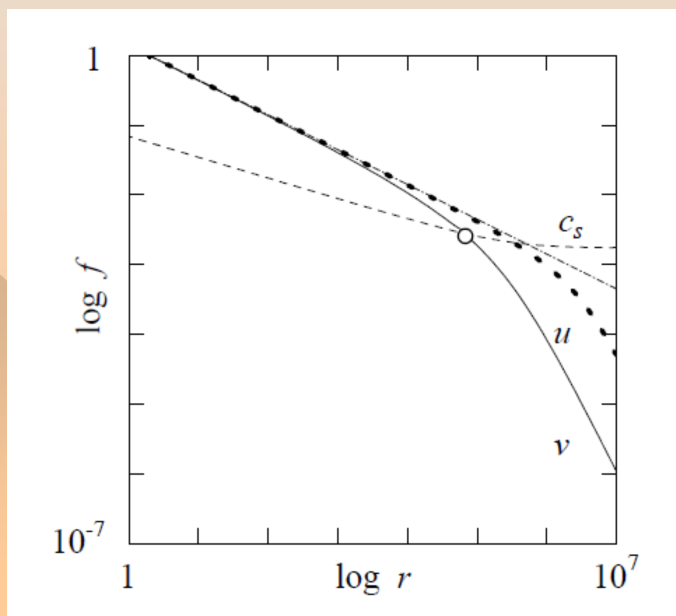
- 速度 v, u
- 相対速度 (u-v)
- 摩擦 f_{drag}

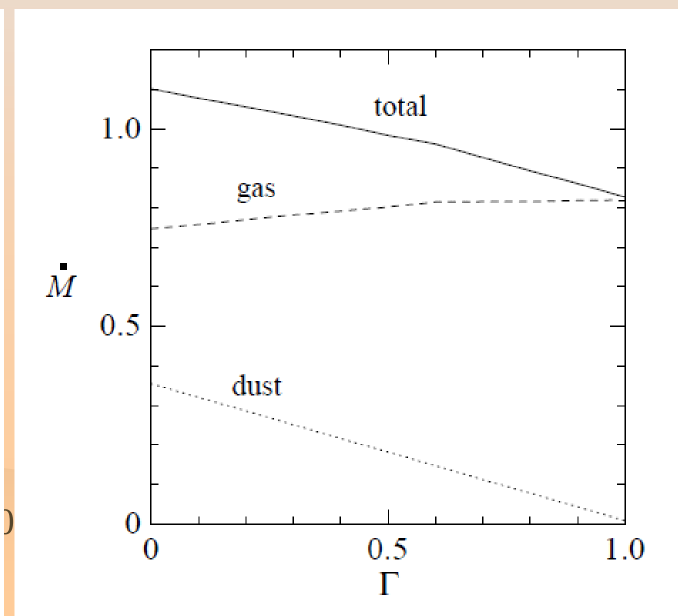
$$v_{\mathrm{drift}} = u - v,$$

$$f_{\mathrm{drag}} = \rho n_{\mathrm{d}} \sigma_{\mathrm{d}} v_{\mathrm{drift}} \sqrt{c_{\mathrm{s}}^2 + v_{\mathrm{drift}}^2},$$

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM}{r^2} + \frac{\sigma_{\mathrm{d}}}{m_{\mathrm{d}}} \nu(u-v)\sqrt{c_{\mathrm{s}}^2 + (u-v)^2}, \quad (9)$$

$$u\frac{du}{dr} = -\frac{GM(1-\Gamma)}{r^2} - \frac{\sigma_{\mathrm{d}}}{m_{\mathrm{d}}} \rho(u-v)\sqrt{c_{\mathrm{s}}^2 + (u-v)^2}. \quad (10)$$





最近の話題と今後の課題

- radiation field, optical depth
- magnetic field
- non-isothermal
- relativistic
- simulation
- young stellar object
- wave
- stability→参考文献
- multi-components
  – dust
  – pair plasma
  – porous media
  – photon bubble

最近の話題と今後の課題

Holzer and Axford 1970; Yalinewich+2018

- Galactic Wind
- stellar wind input
- central mass only
- analytical/sim
- v_{wind}~v_{kepler}で淀み点 stagnation point
- 内側 inflow
- 外側 outflow
- SgrA*

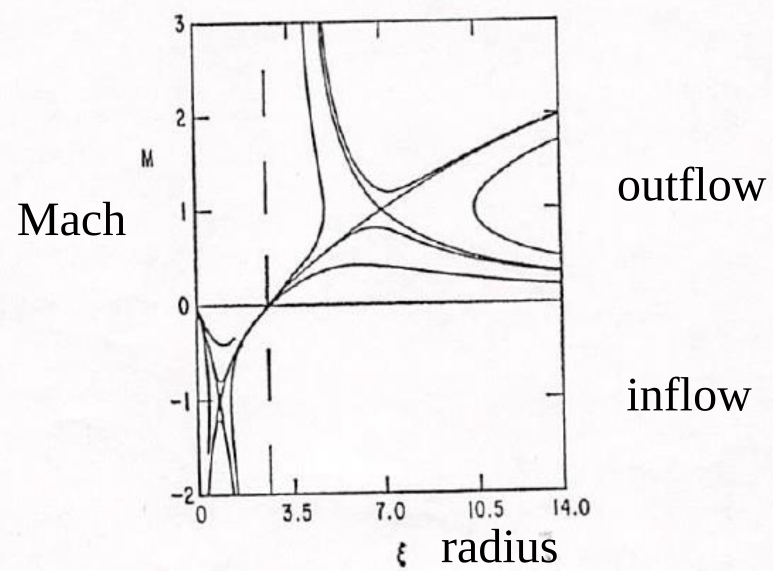


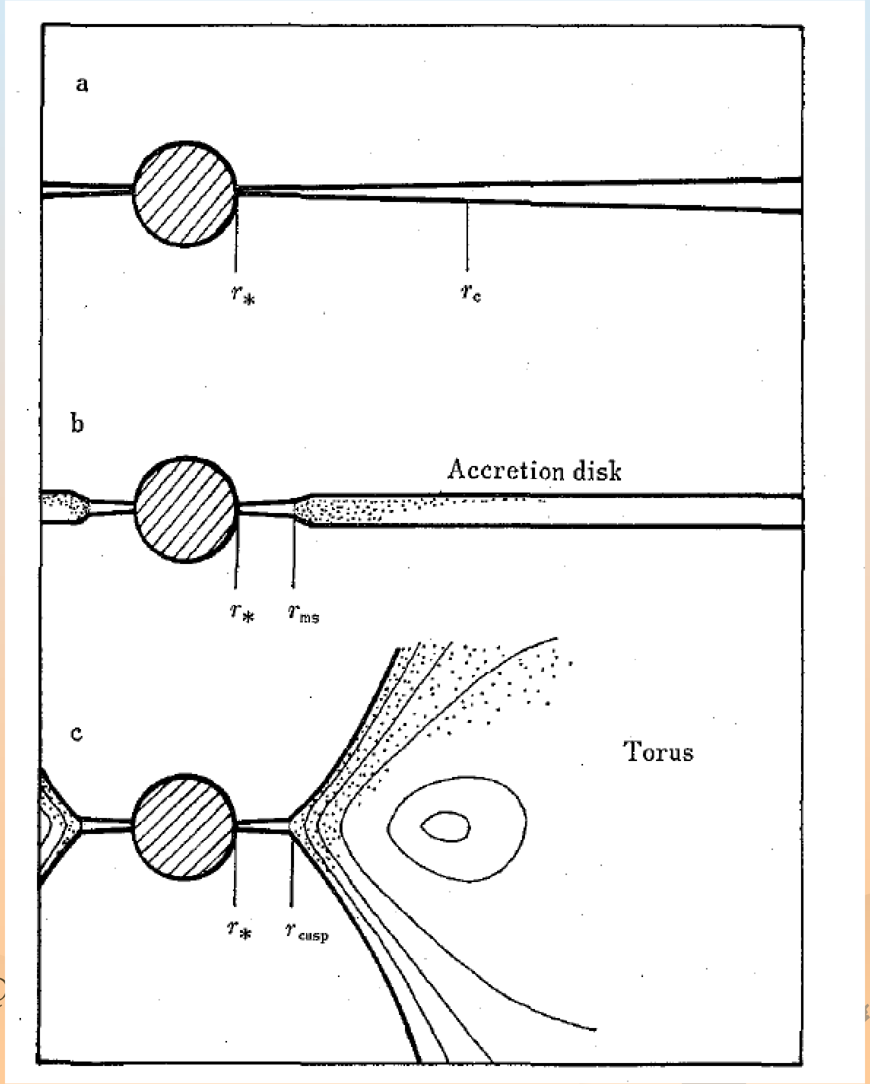
FIGURE 4. Mach number versus radial distance (in units of the scale height of stellar mass density) for the case of a galactic wind allowing for an exponential mass production function and $\gamma=4/3$, $\mu=1$, and $\eta=2$. The stagnation point occurs at $\xi_0(\alpha_0)=3$ with $M>0$ in $\xi>\xi_0$ and $M<0$ in $\xi<\xi_0$. A critical point exists in each of the regions $\xi>\xi_0$ and $\xi<\xi_0$ with one continuous solution intersecting both critical points.

3　円盤降着の遷音速点

Fukue 1987

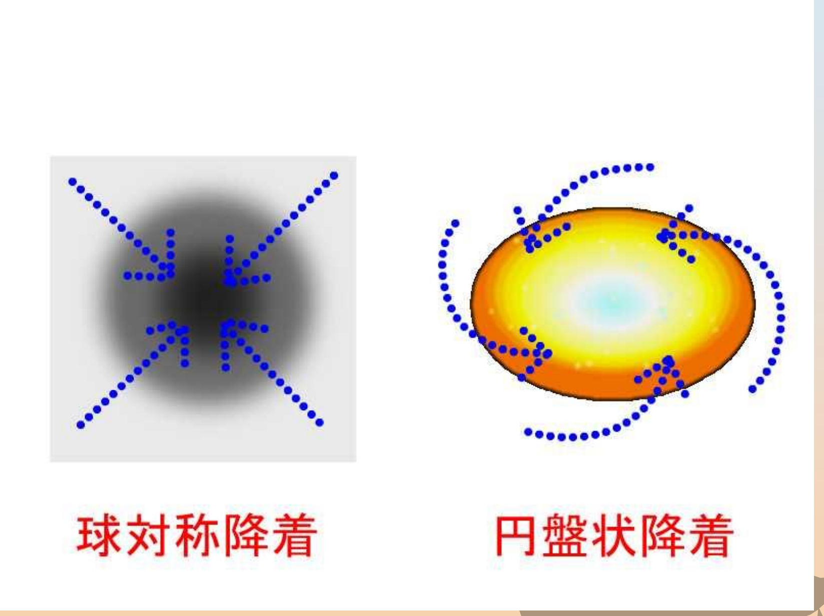
- 重力天体への円盤降着
  - NR（Henriksen and Heaton 1975）
  - GR（Liang and Thompson 1980）
  - GR（Fukue 1987）

- 天文学的事象
  - protoplanetary disk
  - neutron star
  - black hole accretion



円盤降着
仮定

- 円盤降着
  - 質量Mの中心天体
  - 定常
  - 軸対称(r)
  - 角運動量一定
  - 非粘性、断熱
  - 磁場・輻射なし
  - 厳密な状態方程式(vsポリトロピックEoS)



有効ポテンシャル

- エネルギー積分

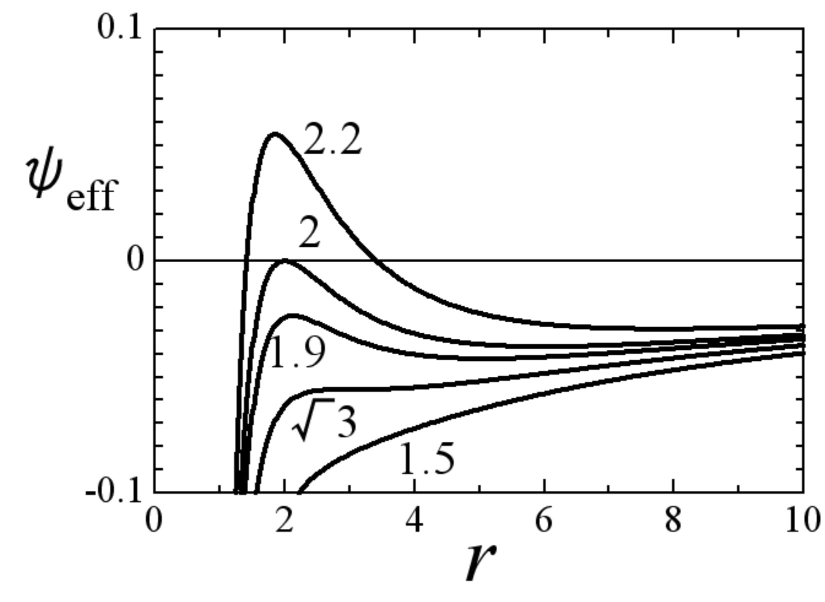
$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + \psi_{\rm eff} = \frac{E^2 - c^4}{2c^2},$$

$$\psi_{\rm eff}(r) \equiv -\frac{GM}{r}\left(1 + \frac{\ell^2}{c^2 r^2}\right) + \frac{\ell^2}{2r^2}.$$

- 有効ポテンシャル
  - 半径r/r_g
  - ポテンシャルψ/c^2
  - 角運動量l/cr_g

2でmarginally bound

√3でmarginally stable



状態方程式

- 圧力 p, 内部E ε
- 温度 T

$$p_a = n_a k_\mathrm{B} T_a, \qquad \text{(D.13)}$$

$$\varepsilon_a = n_a f_a(T_a), \qquad \text{(D.14)}$$

$$f_a(T_a) = m_a c^2 \left[\frac{3k_\mathrm{B}T_a}{m_a c^2} + \frac{K_1(m_a c^2/k_\mathrm{B}T_a)}{K_2(m_a c^2/k_\mathrm{B}T_a)} \right], \qquad \text{(D.15)}$$

$$\frac{df_a}{k_\mathrm{B}dT_a} = 3 + 3\frac{f_a}{k_\mathrm{B}T_a} - \left(\frac{f_a}{k_\mathrm{B}T_a}\right)^2 + \left(\frac{m_a c^2}{k_\mathrm{B}T_a}\right)^2, \qquad \text{(D.16)}$$

where K_n's are the modified Bessel functions of the second kind of order n [3].

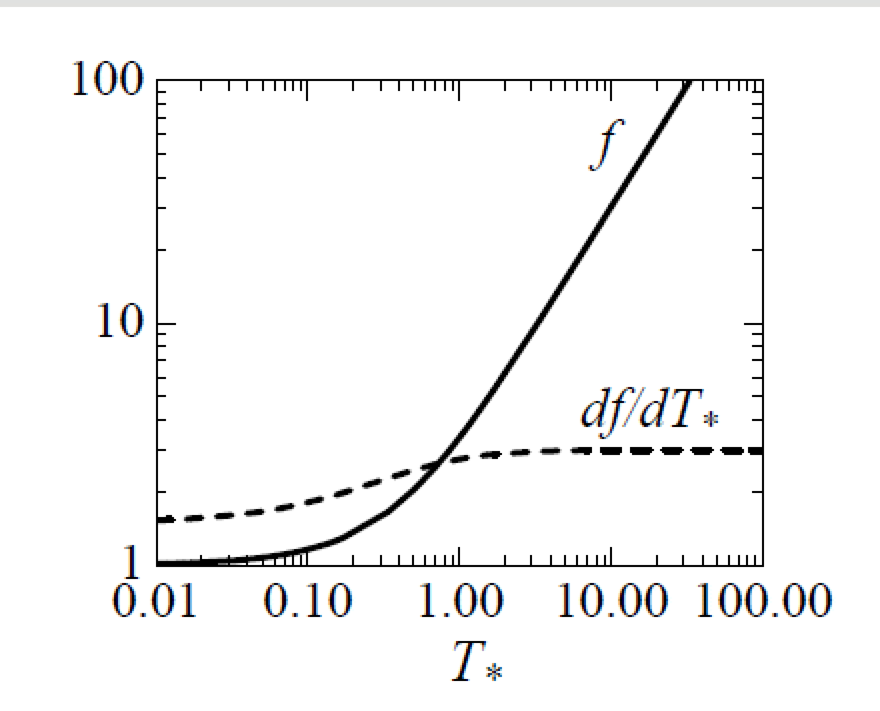
- 音速

$$\frac{c_\mathrm{s}^2}{c^2} \equiv \left(\frac{\partial p}{\partial \varepsilon}\right)_\mathrm{s} = \Gamma \frac{p}{\varepsilon + p}, \qquad \text{(D.20)}$$

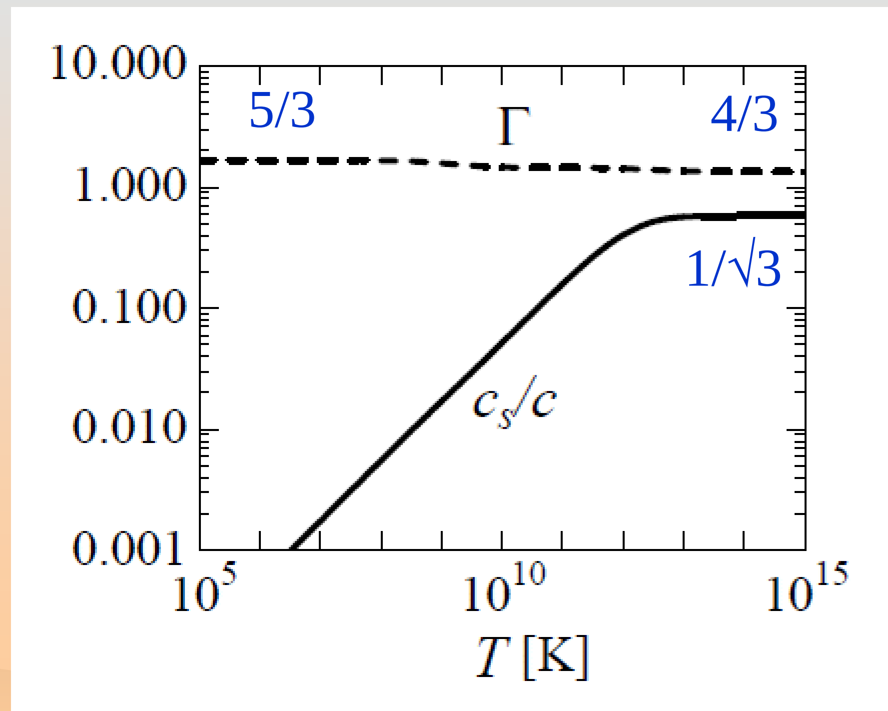
$$\Gamma = 1 + \frac{p}{\sum T_a n_a f_a'(T_a)}, \qquad \text{(D.21)}$$

状態方程式

- 関数f



- 音速、有効Γ



Disk
基礎方程式

- Sch. BH
- 断面積 $A \propto r^N$

$$A n u^r = \text{constant} , \tag{1}$$

$$u^r = \gamma v_r (g_{00})^{1/2} = \gamma_v v (g_{00})^{1/2} , \tag{2}$$

$$u^r \frac{du^r}{dr} + \frac{1}{2r^2} - \left(r - \frac{3}{2}\right) u^\varphi u^\varphi = -\left(1 - \frac{1}{r} + u^r u^r\right) \frac{1}{\varepsilon + p} \frac{dp}{dr} , \tag{3}$$

$$u^r \frac{du^\varphi}{dr} + \frac{2u^r u^\varphi}{r} = -\frac{u^r u^\varphi}{\varepsilon + P} \frac{dp}{dr} . \tag{4}$$

$$\frac{d\varepsilon}{dr} - \frac{\varepsilon + p}{n_p} \frac{dn_p}{dr} = 0 . \tag{5}$$

$$A(\varepsilon + p) u_0 u^r = \text{constant} , \tag{6}$$

$$p = nkT = 2n_p kT , \tag{7}$$

$$\varepsilon = n_p f_p + n_e f_e = n_p (f_p + f_e) , \tag{8}$$

$$(f_p + f_e + 2kT) u_0 = \text{constant} = (m_p + m_e) E , \tag{9}$$

$$r^2 u^\varphi \frac{\varepsilon + p}{n_p} = \text{constant} . \tag{10}$$

$$\frac{r^2 u^\varphi}{u_0} = \text{constant} = L , \tag{11}$$

Disk
風方程式

$$\frac{dv}{dr}=\frac{(1-v^2)v\{c_s{}^2N/r-(1-c_s{}^2)/(2r^2g_{00})+\gamma_L{}^2[1-(3/2r)]L^2/r^3\}}{v^2-c_s{}^2} \tag{12}$$

$$\frac{dT}{dr}=\frac{2T/(f_p'+f_e')\{-v^2N/r+(1-v^2)/(2r^2g_{00})-\gamma_L{}^2[1-(3/2r)]L^2/r^3\}}{v^2-c_s{}^2}, \tag{13}$$

$$c_s{}^2=\left(1+\frac{2k}{f_p'+f_e'}\right)\frac{2kT}{f_p+f_e+2kT} \tag{14}$$

Disk
臨界点

- relativistic
- Newtonian

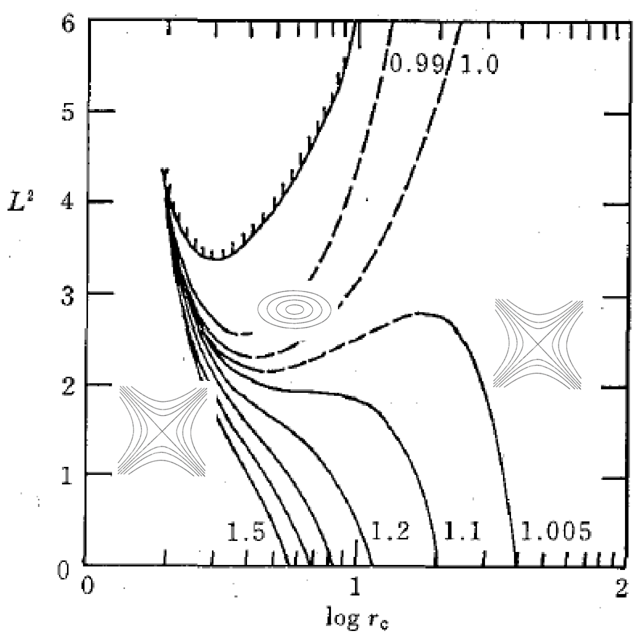


Fig. 2. The relations among L, E, and r_c. The abscissa is r_c and the ordinate is L^2. The number attached on each curve is the value of E (=0.99, 1.0, 1.005, 1.1–1.5). The parameter is $N=2$. It is noted that in some range of E there are multiple solutions r_c for a fixed L. Critical points are of the saddle type (denoted by solid curves) or of the center type (dashed curves). For the steady transonic flow, the values of parameters in the hatched region are forbidden because $v^2|_c<0$ there.

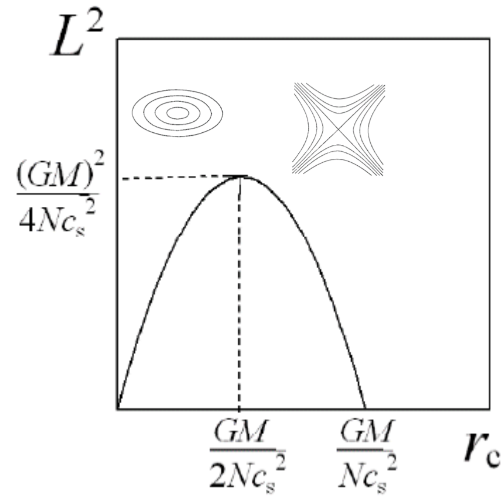


図 6.4 r_c-L^2 平面における臨界点の軌跡.

Disk
遷音速解

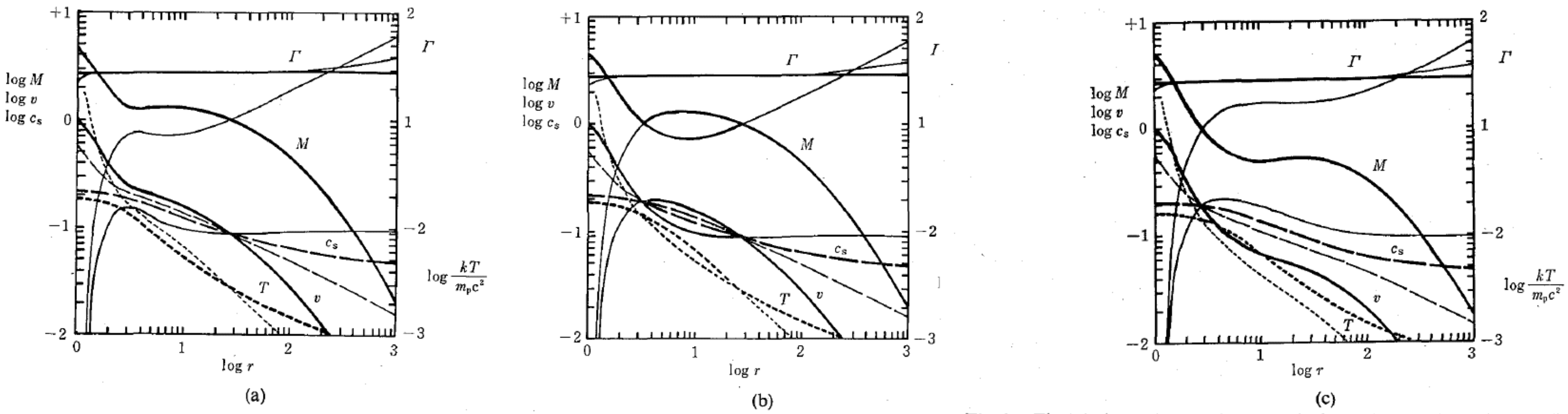


Fig. 3. The Mach number M, the flow velocity v, the sound speed c_s, the flow temperature T, and the effective adiabatic index Γ of typical critical solutions. The parameters are fixed as $N=2$, $E=1.005$, and $L=1.5$ (a), 1.522 (b), and 1.6 (c). Thick curves represent accreting solutions, while thin ones represent wind solutions. The critical solution passes through the outer critical point in (a), whereas it passes through the inner point in (c).

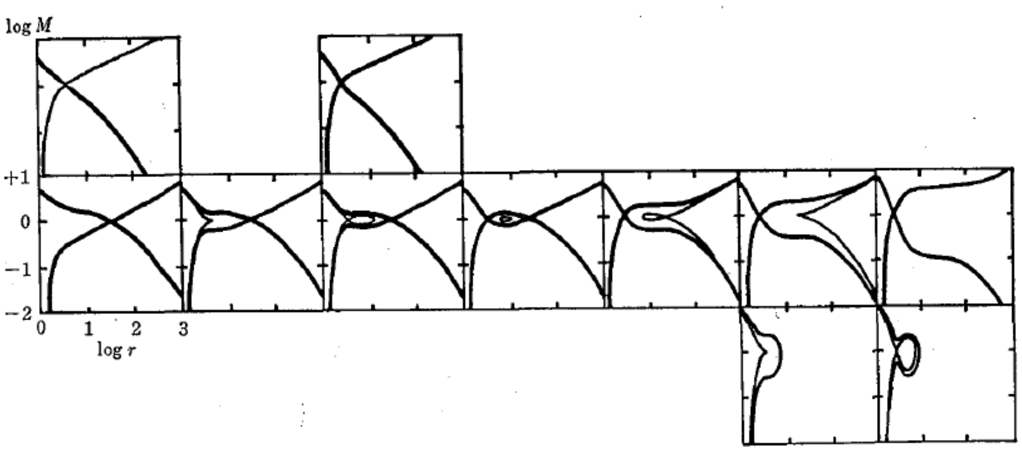


Fig. 5. The Mach numbers for several parameters in figure 4. From top-left to bottom-right, parameters are $(L, E)=(1.20, 1.04)$, $(1.50, 1.04)$, $(1.20, 1.005)$, $(1.46, 1.005)$, $(1.50, 1.005)$, $(1.52, 1.005)$, $(1.60, 1.005)$, $(1.67, 1.005)$, $(1.80, 1.005)$, $(1.71, 0.97)$,

Disk
遷音速解の分類

- 1

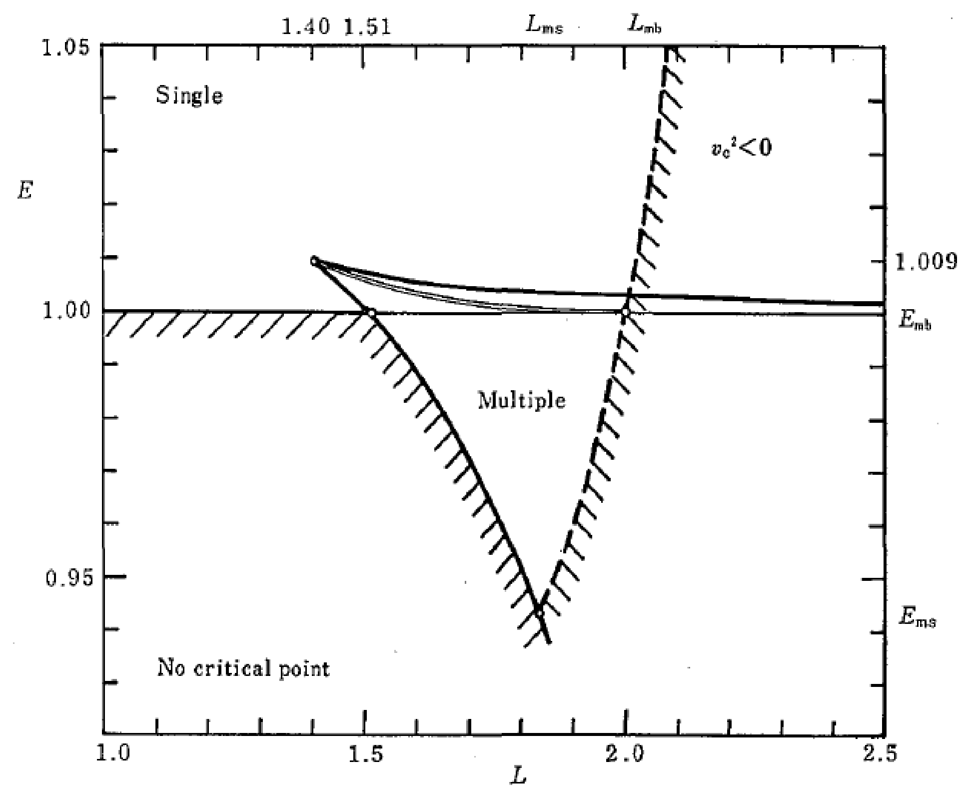


Fig. 4. The classification diagram on the L-E parameter plane. The plane is divided into three distinct regions via the number of critical points. The bifurcation points on the boundary of each domain are $(L, E, r_c)=(1.4047, 1.0087, 6.7)$, $(1.5151, E_{mb}, 4.3)$, $(L_{ms}, E_{ms}, 3)$, and $(L_{mb}, E_{mb}, 2)$, where $L_{ms}=(27/8)^{1/2}$, $L_{mb}=2$, $E_{ms}=(8/9)^{1/2}$, and $E_{mb}=1$. See the text for details.

Disk
円盤降着流中の定在衝撃波

- 跳び条件

$$n_1 u_1 = n_2 u_2 \,, \tag{17a}$$

$$(\varepsilon_1 + p_1) u_1^{\,2} + p_1 = (\varepsilon_2 + p_2) u_2^{\,2} + p_2 \,, \tag{17b}$$

$$(\varepsilon_1 + p_1) \gamma_1 u_1 = (\varepsilon_2 + p_2) \gamma_2 u_2 \,, \tag{17c}$$

$$(f_p + f_e + 2kT)\gamma|_1 = (f_p + f_e + 2kT)\gamma|_2 \,, \tag{18a}$$

$$[2kT + (f_p + f_e + 2kT) u^2]/u|_1 = [2kT + (f_p + f_e + 2kT) u^2]/u|_2 \,. \tag{18b}$$

- エントロピーが増加する
- 安定性

Disk
円盤降着流中の定在衝撃波

- 中性子星
  - 中心でv~0
  - ３か所で可能
- ブラックホール
  - 中心でv~c
  - ２か所で可能

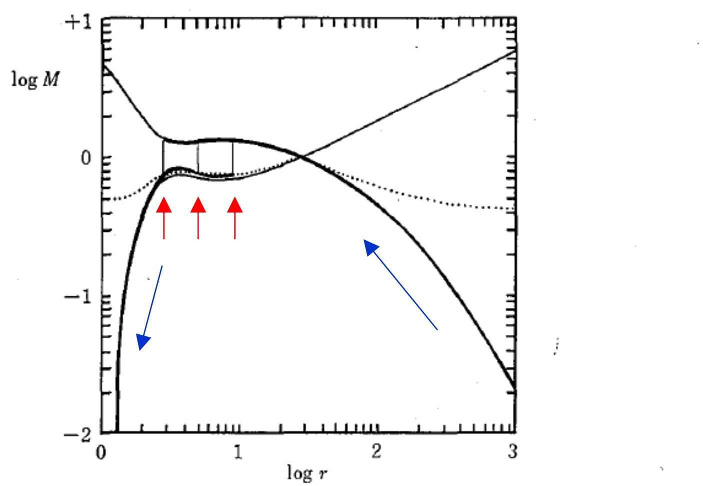


Fig. 6. The Mach number M of an example of solutions with standing shocks in the case of accretion onto a neutron star. Parameters are $(L, E)=(1.500, 1.005)$. The dotted curve denotes the locus of the Mach number just behind the shock. Thin vertical lines represent the positions of standing shocks for the conditions specified. The shocks can stand at three distinct locations for the same parameters and boundary conditions in this case.

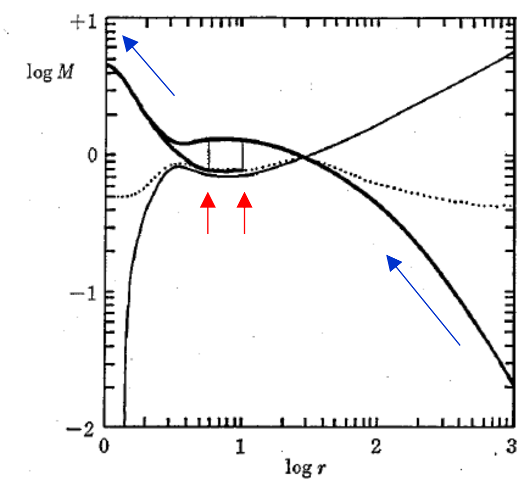


Fig. 7. Same as figure 6 but for a black hole. $(L, E)=(1.513, 1.005)$. In addition to the transonic solution without a shock, there exist two with standing shocks at different radii.

Disk
定在衝撃波の安定性

- Kerr holeの場合
  - Nakayama+JF 1989
- 安定性
  - Nakayama 92, 93, 94
  - Nakayama 95 (rel)
- The flow is unstable if the post-shock flow is accelerated.

- ブラックホール
- Inner shock is unstable
- Outer shock is stable

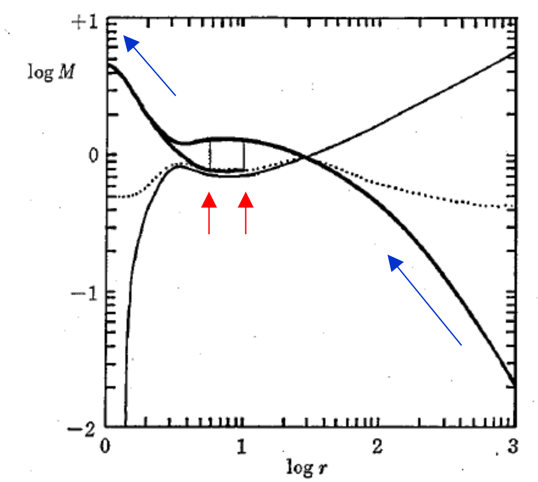


Fig. 7. Same as figure 6 but for a black hole. $(L, E) = (1.513, 1.005)$. In addition to the transonic solution without a shock, there exist two with standing shocks at different radii.

最近の話題と今後の課題

- radiation field, optical depth
- magnetic field
- non-isothermal
- relativistic
- simulation

- wave
- stability
- multi-components
- radiative shocks
- shocked outflow

参考文献

Bondi, H. 1952, MNRAS, 112, 195

Fukue, J. 1987, PASJ, 39, 307

Fukue, J. 2001a, PASJ, 53, 275

Fukue, J. 2001b, PASJ, 53, 687

Fukue, J., Ioroi, M. 1999, PASJ, 51, 151

Henriksen, R.N., Heaton, K.C. 1975, MNRAS, 171, 27

Holzer, T.E., Axford, W.I. 1970, ARA&Ap, 8, 31

Hoyle, F., Lyttleton, R.A. 1939, Proc. Camb. Phil. Soc., 35, 405

Liang, E.P.T., Thompson, K.A. 1980, ApJ, 240, 271

Nakayama, K., Fukue, J. 1989, PASJ, 41, 271

Nio, T., Matsuda, T., Fukue, J. 1998, PASJ, 50, 495

Tejeda, E., Aguayo-Ortiz, A. 2019, arXiv:1906.04923v1 (MNRAS)

Yalinewih, A. et al. 2018, arXiv:1805.03641v2

参考文献

analytical study

Foglizzo, T., Ruffert, M. 1997, A&Ap, 320, 342

Matsuda, T., Isaka, H. Ohsugi, Y. 2015, PTEP, 2015, No11, 0114

numerical simulation

Ohsugi, Y. 2018, Astronomy and Computing, 25, 44

Stability関連

Garlick, A. R. 1979, A&Ap, 73, 171

Kovalenko, I.G., Eremin, M.A. 2002, MNRAS, 298, No3；stable

Vandenbroucke, B. + 2019, MNRAS, 485, 3771-3782；ionized, radiation hydrodynamics

Disk衝撃波

Nakayama, K. 1995, MNRAS, 281, 226